ARISTOTLE, MATHEMATICS, AND COLOUR

INTERMEDIATE COLOURS AS MIXTURES OF BLACK AND WHITE

ARISTOTLE says in the *De Sensu* that other colours are produced through the mixture of black bodies with white (440^{a}31–^{b}23). The obvious mixture for him to be referring to is the mixture of the four elements, earth, air, fire, and water, which he describes at such length in the *De Generatione et Corruptione*. All compound bodies are produced ultimately through the mixture of these elements. The way in which the elements mix is described in 1. 10 and 2. 7. They mix in such a way as to produce an entirely new substance, in which the characteristics of the original earth, air, fire, and water survive only in modified form.

We can guess that elemental fire would be counted by Aristotle as white, and elemental earth as black. For he thinks that it is the presence of fieriness in a body that makes it white, and the absence of fieriness that makes it black.[1] The other two elements, water and air, are usually treated as having no colour of their own[2] (though they can contribute to the colour of a body into which they enter, air making it whiter, and water making it darker).[3]

To our ears, one of the strangest parts of this theory may be the idea that mixtures of black matter with white should produce anything other than grey. Yet Aristotle was not the first to suppose that black and white could produce the other colours. It can be understood why Aristotle and others might *want* to think this. In the simpler case of temperature, many of Aristotle's predecessors had thought of intermediate degrees of temperature as being blends of hot with cold. But why did this conception not appear absurd when it was extended to colours?

[1] *D.S.* 439^{b}14–18, with 439^{a}18–21 and *D.A.* 418^{b}9–20. The fact that fire (especially pure, elemental fire) is to a certain degree transparent may also contribute to its whiteness, in Aristotle's opinion. And the non-transparency of earth may contribute to its blackness. At any rate, on one interpretation *D.S.* 439^{b}6–12 implies that a total lack of transparency would yield black, while a certain degree of transparency increases the whiteness (though, as the next note indicates, too high a degree of transparency will instead make something colourless). Cf. also *G.A.* 779^{b}27–33; 780^{a}27–36 for a connection between non-transparency and blackness.

[2] Thus fieriness in air produces light, not white (*D.S.* 439^{b}14–18). In general, Aristotle (forgetting the case of wine or coloured gems) speaks as if transparent things (like air or water) have no colour of their own, but only borrowed colour, due to reflection or other causes. This is the best interpretation of *D.A.* 418^{b}4–6: 'by *transparent* I mean what is visible, yet not in itself visible speaking without qualification, but visible through borrowed colour.' Cf. also *D.S.* 439^{b}10–14, which, on one interpretation, speaks of 'transparent things themselves, like water and anything else there may be of this kind, I mean those ones which appear to [sc. merely appear to, but do not really] have a colour of their own'.

On the other hand, water and air can have *borrowed* colour due to reflection (*Meteor.* 1. 5 and 3. 2–6; *D.S.* 439^{b}3–6), or other causes (the water of the eye-jelly takes on colour during the act of vision, and this is not due to reflection). Water displays a colour which gets darker, according as the water gets deeper and less transparent (*G.A.* 779^{b}27–33; 780^{b}8).

[3] Air bubbles give whiteness to foam, semen, oil, hair, and other stuffs. What is more watery, and less full of air, is darker (*G.A.* 735^{a}30–736^{a}22; 784^{b}15; 786^{a}7–13).

As a first step towards an answer, we may point to a certain fact about Greek colour words that is documented by Platnauer in his article 'Greek Colour Perception' (*C.Q.* xv [1921], 153). It is that a number of Greek colour words did double duty. They were used as much to denote the brilliance of a colour as to denote its hue. *Leukon* means bright, or light-coloured, as much as it means white. And *melan* means dark-coloured, as much as it means black.[1] Now Aristotle's theory will not seem quite so bizarre if we think of red as a bright colour, and if we think of him as saying that it is produced by mixing a lot of the brightest colour with a little of the darkest.

This is both similar to and different from what is said in modern colour theory. In the systems of Munsell and of Ostwald, coloured chips are placed in a three-dimensional arrangement. In one dimension, the chips vary in respect of brilliance, ranging between black at one extreme and white at the other. Intermediate brilliances can be specified in terms of such and such a percentage of white and such and such a percentage of black. So much is reminiscent of Aristotle. But it is not, of course, the case that red has only one brilliance, nor that there is any brilliance that attaches only to red.

In our search for an explanation, we ought to make use of a second consideration, and observe how Aristotle reaches his theory of the mixture of black with white, or at least how he tries to lead us to the theory. In *De Sensu* 3, he starts from two theories which he rejects, the theories that black and white produce the other colours by being juxtaposed with, or superimposed upon, each other. Aristotle leads us to his theory by substituting for juxtaposition and superimposition his pet notion of chemical mixture. This substitution is presented as enabling us to avoid the difficulties in the other two theories.[2] So it is made to look as if the Aristotelian theory has grown out of the other two. But why had the other two theories appealed to anyone? Aristotle speaks as if the grounds for them were *a priori*. A combination of black and white can't appear exclusively black, or exclusively white, but must have some colour; so it has an intermediate colour (*D.S.* 439^b22–5; 440^a24–6). We may wonder why not grey. But there is a piece of empirical evidence which Aristotle mentions in connection with the theory of superimposition. The sun is white (*leukos*), but if we look at it through a cloudy or a sooty medium, its appearance is darkened to red.[3] In this example, a reduction in brilliance goes hand in hand with a change of hue. And both are produced by superimposing black, sooty particles over a white light. Perhaps this phenomenon encouraged adherents of the superimposition theory. It certainly encouraged Goethe many centuries later, when, in the section on physical colours in *Zur Farbenlehre*, he described physical colour as involving a relationship between light and darkness. The case of the sun obscured by atmospheric conditions is one of the cases he cites. His theory is

[1] For symptoms of these facts in the *De Sensu*, see 439^a18–19 'light [i.e. illumination] is the colour of the transparent'; 439^b2 'sheen is a sort of colour'; 440^a11 'the sun appears *leukos*'.

[2] Both theories appeal to an illusion. When close up, one would see that there were two colours, black and white, instead of one. The theory of juxtaposition tries to avoid this by making the black and white specks too small to see, but this is a further impossibility. Also against the juxtaposition theory is that it is linked with the untenable idea that effluxes stream from the seen object into one's eyes, and misguidedly postulates a time-lag too short to perceive between the arrival of the black and of the white particles (*D.S.* 440^a15–31, ^{b}16–18).

[3] *D.S.* 440^a7–12. Aristotle mentions similar phenomena in the *Meteorology* (374^a3; ^{a}27; ^{b}10).

often compared to Aristotle's, but it is, in this respect, more like the superimposition theory, which Aristotle rejects.[1]

No empirical support is mentioned in connection with the theory of juxtaposition. And it is not particularly likely that any would have been available to the inventors of the theory, though there is a phenomenon which they might have used for support, if only they had known of it. The English painter, Bridget Riley, has produced pictures in which black and white are juxtaposed, in long ribbons, not in imperceptible specks. When people look at these black and white ribbons, many are able to see all sorts of colours appearing. A similar use was made of black and white dots in early attempts to produce colour television. And the principle is exploited in an optical instrument known as a reflection grating. Whether the effect is sufficiently common in nature to have influenced the juxtaposition theorists is extremely doubtful.

The Mathematical Distinction of Shades

It is in connection with the mixture of black and white that Aristotle introduces mathematics into his theory. He hopes to apply to colours and flavours the mathematical treatment that had recently proved so successful in acoustical theory. Consonant notes in music, according to the definition most commonly given by Greek writers, are ones which blend together when played simultaneously.[2] They were also considered the most pleasant combinations. Dissonant notes, on the other hand, compete with each other, so that each is heard separately. Notes separated by an interval of a fourth, a fifth, and an octave were the first to be recognized as consonant. But Aristotle is aware that there are other consonant pairs as well (*Metaph.* 1093^a26). To produce two notes an octave apart, one can pluck a string, halve its length, and pluck it again. Thus the string-length ratio corresponding to the interval of an octave is 2:1. Other consonant pairs also turned out to have uncomplicated ratios, namely 4:3 (fourth), 3:2 (fifth), 3:1 (octave+fifth), 4:1 (double octave). All these ratios are expressible by the numbers 1 to 4. Three of them are of the form n:1 (4:1, 3:1, 2:1). And three are of the form n+1:n (4:3, 3:2, 2:1). Aristotle never mentions the awkward exception, the octave+fourth, whose ratio is 8:3.

The extension of these mathematical ideas to colour and flavour is made in the *De Sensu* 3, 4, and 7. The treatise *De Coloribus* expresses a quite different theory, and I follow the normal view in taking it to be un-Aristotelian. The theory of the *De Sensu* is that colours intermediate between the darkest (black) and the lightest (white) consist of a mixture of black with white. And the most

[1] Goethe (*Zur Farbenlehre*, 1810, translated with notes by Eastlake as *Goethe's Theory of Colours*, John Murray, 1840), knew, and wrote about, Aristotle's theory, but he mistakenly counted as Aristotelian the *De Coloribus*, which talks in ch. 2 of mixing lights, not stuffs.

For details of modern colour theory, consult Ralph M. Evans, *An Introduction to Colour*, John Wiley and Sons, 1948, esp. pp. 65–6, 68–9.

[2] Archytas' followers, *ap.* Porphyry's commentary on Ptolemy's *Harmonics*, ed. Wallis, p. 277; Plato, *Timaeus* 80A–B; Aristotle, *D.A.* 426^b3–6, *Metaph.* 1043^a10 (cf. *D.S.* 447^a12–448^a19); pseudo-Aristotle, *Problems* 19. 27, 38; Euclid, *Sectio Canonis*, introd., ed. Meibom, p. 24; pseudo-Euclid, *Isagoge* § 5, ed. Meibom, p. 8; Nicomachus, *Enchiridion* § 12, ed. Meibom, p. 25; Gaudentius, *Isagoge* § 8, ed. Meibom, p. 11; Aelian, *ap.* Porphyry, loc. cit., pp. 218, 265, 270; Boethius, *De Institutione Musica*, 1. 3, 1. 8, 1. 28.—In general, see F. A. Gevaert, *Histoire et Théorie de la Musique de l'Antiquité*, Ghent, 1875.

pleasant colours are produced by ratios of black to white which are uncomplicated,[1] and ($439^{b}32$) expressible in numbers easy to calculate with. They are thus parallel to consonant combinations in music. The other colours are not *logoi*, or uncomplicated ratios ($439^{b}29$–30; $440^{a}14$–15). Indeed, they are not expressible in (rational) numbers at all ($440^{a}2$–3; $440^{b}20$; $442^{a}15$–17), but stand in an incommensurable ($439^{b}30$) relation of predominance and subordination only ($439^{b}30$; $440^{b}20$).

Aristotle does at $440^{a}3$–6 mention an alternative distinction, according to which all colours are expressible in (rational) numbers, but some are regular and some irregular.[2] He does not, however, refer to this alternative again, except perhaps at $442^{a}14$–16.[3] Otherwise he mentions it only in connection with an idea he rejects, that the mixture of black and white takes the form of a juxtaposition of tiny particles of black and white. When he goes on to give his own view that the mixture of colours is due to genuine chemical mixture of coloured bodies, not to a mere juxtaposition, he repeats only the original distinction between the rational and the irrational combination.

We may guess that this mathematical idea in *De Sensu* 3 fits in with Aristotle's division of colours into three main groups in chapter 4. First of all, there are black and white, the primary colours. And with these, he says, either yellow or grey should be counted in ($442^{a}21$–3; $448^{a}6$).

[1] The word *logos* sometimes implies uncomplicated ratios (*An. Post.* $90^{a}19$; 20; *D.S.* $439^{b}27$–$440^{a}3$; $440^{a}14$–15; *Probl.* 19. 41). But sometimes it has a wider sense, covering all sorts of mathematical relations (*An. Post.* $90^{a}22$; *D.S.* $440^{b}19$; $442^{a}15$; $448^{a}8$; $^{a}10$), even incommensurable ones.

[2] A major problem about $440^{a}3$–6 is that it starts off by suggesting that all colours are (expressible) in (rational) numbers, but appears to finish up by talking of a sub-class which are not in numbers. If we retain the traditional text, the best way to remove this appearance of contradiction is probably that of J. I. Beare, in the Oxford translation. Instead of 'not in numbers', we must understand Aristotle to mean 'not such (i.e. not pure) in their numbers'. I would translate: 'Or one can also suppose that all colours are in numbers, but some are regular, others irregular, and these latter are produced when the colours are not pure through not being pure in their numbers.' This, admittedly, is a strain on the Greek word order. In this translation, I take it that the impure colours are not a sub-class of, but are identical with, the irregular colours. It is for one and the same reason that they are called 'irregular' and 'impure'. For the meaning of 'impure', see below, p. 297.

[3] $442^{a}14$–16 says 'according to a *logos* [or?] in a relation of more and less, whether according to certain numbers in the mixture and interaction, or also in an indefinite way'. I prefer to stick to the MSS. reading, which omits 'or'. In that case, the opening phrase, 'according to a *logos* in a relation of more and less', will be a perfectly non-committal one, which does not specify the particular relationships available. It need mean no more than 'in a quantitative relationship'. The insertion of 'or' has appealed to those who over-hastily connected the word *logos* with the *logoi*, or *uncomplicated* ratios, of chapter 3 ($439^{b}29$–30; $440^{a}14$–15), and the words 'more and less' (*mallon kai hêtton*) with what in chapter 3 is described as *merely* (*monon*) an *incommensurable* (*asummetron*) relation of predominance and subordination (*huperochê kai elleipsis*, $439^{b}30$; $440^{b}20$).

On our interpretation, it is left to the following words to specify what the possible relationships are. And the following words can be taken in either of two ways. Perhaps the phrase 'whether according to certain numbers in the mixture and interaction' introduces the second and less usual alternative of $440^{a}3$–6, according to which all the relationships are commensurable and rational. The last phrase ('or also in an indefinite way') will then revert to the more usual alternative, according to which some relationships are commensurable, but there are 'also' incommensurable ones.

Alternatively, the remaining words confine themselves to the more usual alternative, and simply spell out the choice it offers between being in rational numbers and not being in rational numbers. In that case, the less usual alternative is never alluded to again after its original mention.

The second group consists of the secondary colours, red, purple, green, and blue, and possibly yellow (442ª20–5; 448ª8). These are the result of direct mixture of black with white. But the mixture is naturally, not artificially, produced. In fact, these colours, or the first three of them, are listed in the *Meteorology* (372ª2–9) as the ones that painters cannot get by mixing. We may guess that these secondaries are the colours that are compared with consonant combinations of sound. At any rate, *De Sensu* 3 specifies purple, red, and a few like these as corresponding to the consonances, and as having a ratio expressible in rational numbers easy to calculate with (440ª1).

The third group consists of the tertiary colours, which are mixed out of these (442ª25), i.e. presumably out of the secondary colours, instead of being mixed directly out of black and white.[1] Of their original ingredients, therefore, one, e.g. blue, will exemplify one ratio, another, e.g. red, will exemplify another. It seems just possible that this is what Aristotle means when he alludes briefly to the alternative method for distinguishing colours, according to which some colours are irregular and impure (440ª3–6). Perhaps 'impure' means that the original ingredients exemplify more than one ratio, and not, as Alexander of Aphrodisias says, that the final product exemplifies more than one.

Be that as it may, we can guess that the tertiary colours are the ones which correspond to dissonant notes, and which Aristotle normally classifies not as irregular and impure, but rather as having irrational ratios. His usual view of tertiary colours is that, like secondaries, they have a single ratio of black to white (see especially 448ª10, 'in this way the ratio of the extremes becomes single').

How much of the foregoing scheme is Aristotle's and how much did he inherit? Oskar Becker[2] has suggested that the rational/irrational division stems from Archytas and Eudoxus, while the alternative regular/irregular division is due to Philolaos, Plato, and the Old Academy. Konrad Gaiser[3] thinks that such precision is impossible, but that the mathematical ideas were already being worked on in the Academy before Aristotle wrote about them. A. E. Taylor[4] detects a Pythagorean source. In fact, it is hard to say how much is due to Aristotle. He certainly learnt from others the theories that the remaining colours are produced from black and white by juxtaposition or by superimposition, while the substitution of chemical mixture for juxtaposition and superimposition is his own. But what about the introduction of a threefold division of colours in place of the twofold division of Empedocles, Democritus,

[1] One would suppose, however, that a tertiary colour, such as pink, could be produced not only by mixing two secondary colours, but also by mixing one secondary colour (red) with white, or with colourless water, or again by mixing two tertiary colours.

[2] 'Eudoxos-Studien V', in *Quellen und Studien zur Geschichte der Mathematik*, Abt. B. 3, 1936, p. 403.

[3] 'Platons Farbenlehre', in *Synusia*, Festgabe für Wolfgang Schadewaldt, edd. Hellmut Flashar and Konrad Gaiser, Pfullingen, 1965.

[4] *A Commentary on Plato's Timaeus*, Oxford, 1928, pp. 485, 489, 491. More extravagantly, J. Zürcher claims that the theory is not Aristotle's, but was added later by Theophrastus, under the influence of Aristoxenus (*Aristoteles' Werk und Geist*, Paderborn, 1952, pp. 302–5). A comparatively extensive contribution by Aristotle seems to be allowed in the account of P. Kucharski, 'Sur la théorie des couleurs et des saveurs dans le "De Sensu" aristotélicien', *Revue des études grecques* lxvii (1954), 355, and F. M. Cornford, 'Mysticism and Science in the Pythagorean Tradition', *Classical Quarterly* xvi (1922), 144.

and Plato?[1] What about the idea that the ratios will correspond to certain acoustical ones? What about the idea that the relevant distinction is that between uncomplicated ratios and irrational ones? What about the alternative idea that, if all the ratios are rational, then the distinction must be in terms of regular and irregular? Fortunately, we need not decide how much of this Aristotle is inventing and how much he is merely endorsing. For it will make no difference to our subsequent discussion of his responsibility for errors and miscalculations. Nor will it affect our discussion of whether he thinks it appropriate to apply mathematics to natural science.

Apparent Oversights

We must now observe that unfortunately Aristotle's exposition contains a number of apparent oversights. I shall bring forward five of them for discussion. The first is a minor one, since it involves no error on Aristotle's part, but only an omission. It concerns the question of why painters can't obtain secondary colours by mixing. Mathematically speaking, this ought to be possible. For given three vats of paint, each vat containing black and white in a different ratio, one should be able, as far as mathematics is concerned, to mix a suitable amount from the darkest vat with a suitable amount from the lighest, and obtain the ratio that yields the intermediate shade. Why, then, cannot one obtain a secondary colour by mixing together a darker one and a lighter one in suitable quantities? Indeed, why cannot one obtain any shade by mixing together any darker one and any lighter one in the right amounts? The answer cannot lie in mathematics.

Presumably, the material pigments are recalcitrant in some way. But in what way? It is a matter for regret that Aristotle does not discuss the divergence between the mathematical, and the real, possibilities. One explanation would be that certain pigments, just like oil and water, will not mix. Or at least, the techniques we have so far tried, such as stirring, will not make them mix. Alternatively, perhaps some pigments mix in such a way that, instead of getting a single colour, green, at the end of the mixing process, we get two colours, some brown and some orange. Another explanation would be that the pigments available as ingredients for our mixture are never perfectly homogeneous, and do not display a single ratio throughout.

There is yet another suggestion that we should mention, if only to get it out of the way. It is that when the material pigments are recalcitrant, this is because the ratio in the final mixture is not a function of the ratio in the original ingredients. In the *De Sensu* at 440^b19, Aristotle seems to be thinking about the ratio of black to white in the original ingredients, in the original earth and fire, for example, before they were mixed together. But the *De Generatione et Corruptione*[2] suggests a sense in which black and white not only attach to the original ingredients, but also persist in modified form in the resulting compound colour. For the resulting purple or brown has a certain degree of darkness (or black) about it, and a certain degree of brightness (or white). So instead of talking of the ratio of the original black to the original white, one could talk of the ratio of the resulting modified black to the resulting modified white. This

[1] Empedocles, in Aëtius I. 15. 3; Democritus in Theophrastus, *De Sensibus* 73–82, and in Aëtius I. 15. 8; Plato in the *Timaeus* 67C–68D.

[2] See 327^b22–31; 328^a29–31; 334^b8–30, with Harold H. Joachim's useful commentary, *Aristotle on Coming-to-be and Passing-away*, Oxford, 1922.

way of talking is used, in connection with hot and cold, not black and white, at *G. et C.* 334^b14–16. And on the present suggestion, what happens when the material pigments are recalcitrant is that the ratio of modified black to modified white in the compound colour is not a function of the ratio of black to white in the original ingredients. Fortunately, we need not pin this idea on Aristotle, since he never expresses it. It would make his theory less attractive in several ways. We started with one unexplained piece of terminology, the ratio of black to white in the original ingredients. But now we should have a second piece, the ratio of modified black to modified white. How would these ratios be measured? Secondly, we should have a very large gap in our explanation of how a given shade is produced, if the final ratio were not a function of the original ratio.

The second apparent oversight in Aristotle's mathematical theory is this: he seems to ignore the middle ground between the very simplest ratios and those which cannot be expressed in rational numbers at all. Within this middle ground there fall, for example, 9:8 and 256:243, the non-consonant intervals of a tone and a semitone. Both are discussed by Plato at *Timaeus* 36B, where Plato points out that the ratio of 256:243 is in rational numbers (*arithmos pros arithmon*). In fact, there must be an infinity of string-length ratios which fall within the middle ground (even though Aristotle would not allow that, in any but a very weak sense, one could perceive the difference between one ratio and another that was exceedingly close to it).[1] We shall be very surprised, then, when we read *Posterior Analytics* 90^a21–2, and find that Aristotle equates the question 'Is it possible for the high and the low note to be consonant?' with the question 'Is their ratio expressible in (rational) numbers?' This equation implies that any ratio which can be expressed in rational numbers at all will yield a pair of consonant notes. And this is not correct. It is only certain rational ratios, in most cases very simple ones, that yield consonant pairs. The ratio 256:243 does not. We can hardly believe that a member of Plato's Academy was unaware of these facts. What then, is the explanation of Aristotle's overlooking them? We get the same omission in the account of colour in the *De Sensu* (439^b27–440^a3). After describing the simplest ratios, and assigning them to the few pleasantest colours, Aristotle says at 440^a2, 'The colours which are not in numbers,[2] one may suppose, are the other ones.' This again implies that the only alternatives are having very simple ratios, or having ratios that cannot be expressed in (rational) numbers at all.

There are various reflections that ought to have saved Aristotle from this apparent oversight. One, of course, would be to think about non-consonant intervals in music, like 256:243. But in addition to this, mathematics makes it unlikely that all tertiary colours will have irrational ratios. For mathematics suggests that certain quantities of blue, with its uncomplicated ratio, mixed into certain quantities of red, with its uncomplicated ratio, will yield a product whose ratio of black to white is itself uncomplicated, and expressible in

[1] *D.S.* 445^b31–446^a20. Tiny variations cannot be perceived on their own, but are perceived only through being part of, and through contributing to, a larger variation.

[2] 'Not in numbers' (440^a2–3) must mean 'not expressible in (rational) numbers at all'. We cannot rescue Aristotle by taking it to mean 'not in *simple* numbers'. The next line (440^a3–4) rules out this interpretation, by putting forward the alternative that *all* colours are 'in numbers'. This cannot mean 'in *simple* numbers', for there are not enough simple ratios to go round *all* the colours.

rational numbers. At least, this will be so if we set aside the various ways in which the actual material pigments may be recalcitrant. Another warning is supplied by considering further the rather imperfect analogy between colour and sound. If a mixture of red and blue corresponds to anything in music, it corresponds to a combination of two different consonant pairs played simultaneously. Now the latter may be such that all four of its notes are consonant with each other, and bear to each other uncomplicated string-length ratios, expressible in rational numbers. Why then should a mixture of red and blue inevitably result in an irrational ratio?

Our third complaint is this. Aristotle has failed to observe that there will be twice as many simple ratios available for colours as for sounds. This can be seen from the following table.

Shorter string		Longer string	White		Black
1	:	4	1	:	4
1	:	3	1	:	3
1	:	2	1	:	2
2	:	3	2	:	3
3	:	4	3	:	4
			4	:	3
			3	:	2
			2	:	1
			3	:	1
			4	:	1

The reason is that in acoustics, the lower number in a ratio, such as 1:2, must always correspond to the shorter string. But in colour theory, the lower number may correspond *either* to white *or* to black. This has serious consequences for Aristotle's theory. For he says that pleasant colours and consonances are few in number, and that this is for the same reason, namely that there are only a few simple mathematical ratios available.[1] This claim is now spoilt by the fact that there are at least ten simple ratios available for colours, twice as many as for sounds. We may wonder why he does not recognize ten pleasant colours, corresponding to the ten simple ratios we have listed in the table above. But if the pleasant colours, described as 'purple, red, and a few like these' (440^{a}1), are the secondary colours, there will be only five of them. Indeed, there are only seven colours altogether, according to chapter 4, though this statement ignores the tertiary ones. If there are five pleasant colours, the number of mathematical ratios (ten) cannot after all account for their scarcity. In addition, we are left with an unresolved question: which of the ten simple ratios enter into the five pleasant colours?

For our fourth and fifth complaints, we move on to a passage in *De Sensu* 7. But we must treat this passage with more caution, because Aristotle is not endorsing the argument which he propounds. Indeed, he thinks its conclusion mistaken. This is not to say that he is describing the argument merely for polemical purposes, or trying to make it sound silly. On the contrary, the argument is one of a set of three which he thinks plausible enough to create a genuine *aporia*, and worth propounding at length. But his main concern is to press on to his own view (449^{a}5–20). He does not trouble to point out errors in any of the

[1] 440^{a}1–2.

three arguments on the other side. We can hardly suppose that he is unaware of any. We should not therefore attach significance to oversights in the present argument, unless either they have been gratuitously introduced, or they entirely remove the argument's plausibility. With these warnings, we may now look to see how the argument at $448^{a}1$–13 goes.

The argument is trying to establish a conclusion that Aristotle will ultimately reject, namely that one cannot perceive two sense-objects simultaneously. One cannot perceive black and white simultaneously, so the argument alleges, because, being contraries, black and white set up contrary processes in a perceiver, and contrary processes cannot exist in a single place at a single time ($448^{a}1$–5). The same applies to grey and yellow, for grey is a kind of black and yellow a kind of white ($448^{a}5$–8; cf. $442^{a}20$–3). The same applies to red and purple ('mixed' colours, $448^{a}8$),[1] for these two are opposed to each other in a way. For one contains much black and little white, the other the opposite. Or one contains an odd number of units of black to an even number of white, the other the opposite. The only case in which one can perceive red and purple simultaneously is when they are mixed with each other to form a single intermediate colour which one sees as single. Then there is a single ratio of black to white, and so no opposition ($448^{a}8$–13), and so no barrier to one's perceiving the red and purple simultaneously.

I have stated the argument in terms of the colours, red and purple, because it is true of these, on Aristotle's theory, that if one mixes them with each other to obtain an intermediate colour, the resulting intermediate colour will have a single ratio of black to white. But the example actually given is drawn from the field of sound. And our fourth complaint is that the argument has failed to notice that there is no corresponding possibility here. For if we play simultaneously the two notes that constitute an octave and the two notes that constitute a fifth, we shall not thereby obtain a single string-length ratio. There will still be two ratios. This oversight seems rather elementary. But is it the fault of Aristotle or of the argument? The argument cannot afford to admit that there is ever simultaneous perception, when there are two opposed ratios. It can avoid admitting this, by denying that we ever hear an octave and a fifth with perfect simultaneity. There is no need, then, for Aristotle to make the argument take the alternative way out, of claiming that there will be a single string-length ratio. Aristotle appears to have introduced the error gratuitously.

[1] 'Mixed' is a confusing word to use, since in one sense, black and white are mixed colours, i.e. colours that mix *with each other* to form other colours. But Aristotle clearly means to refer to colours like red and purple, which are mixed in the sense that each is composed of black and white.

'Mixed things' cannot refer to black and white. For (a) The case of black and white has already been dealt with in $448^{a}1$–5. (b) Lines 11–13 mention not two terms, but four (much: little and little: much). These four terms must correspond to the black and white that enters into a purple and the black and white that enters into a red. (c) If it were black and white that Aristotle was describing as 'mixed', he would be implying that they were already mixed with each other to form a single intermediate colour. He could not then go on to say ($448^{a}8$) that it would be impossible to perceive them simultaneously.

I believe there will be no obstacle to giving 'mixed' the required reference, provided we make a small emendation of the text at $448^{a}8$–9 from λόγοι ἀντικειμένων to λόγοι ἀντικείμενοι. With this emendation, the argument will be using the opposition *between* red (which is one ratio) and purple (which is an opposed ratio), to explain why one can't perceive red and purple simultaneously. Without the emendation, Aristotle will be irrelevantly emphasizing the opposition *within* red and *within* purple.

The fifth and last oversight on our list enters into the same argument. It has not been gratuitously introduced, however, and we cannot attach equal significance to it. But we may mention it, for the sake of completeness. The argument asks us to suppose that purple contains much black and little white, and red the opposite; or that one contains an odd number of units of black to an even number of white, the other the opposite. But this neglects the fact that many combinations (whether in the field of colour or of sound) will exhibit neither kind of arrangement. Indeed, the first kind of arrangement (much to little and little to much) is impossible in the case of sound, from which the argument draws its illustration. For here the higher number (or, in other words, the much) always corresponds to the greater string-length, and so to the lower pitch. It is not like the case of colour, where the higher number may correspond either to black or to white. Consequently, we shall never get instances of much high pitch to little low pitch. For this would mean, *per impossibile*, that the higher number (the much) corresponded to the higher pitch. The example actually given is that of the octave and the fifth. The ratios here are 2:1 and 3:2. And presumably, the argument would say that this exhibits the second kind of arrangement, even to odd, and odd to even. This assumes that 1 can be counted as an odd number. Certainly, Plato so counts it, at *Phaedo* 105C and *Hippias Major* 302A. So it will not be altogether surprising if the present argument does the same. But there is something a little unsatisfactory about its counting 1 as an odd number. For 1 lacks some of the properties by which odd numbers were commonly defined, such as having a middle, or being divisible into two unequal sets of integers. Moreover, Aristotle himself does not count 1 as a number (*Metaph.* $1088^{a}6$), whereas he does treat oddness as a property peculiar to numbers (*Metaph.* $1004^{b}10$–11; $1031^{a}1$–6).

Aristotle's Willingness to Apply Mathematics to Nature

We have now finished expounding the mathematical aspect of Aristotle's theory. And I should like to raise, in relation to this aspect of his theory, three questions about his method in natural science. The first question concerns his willingness to use mathematics. It is tempting to contrast Plato as one who applies mathematics to natural science with Aristotle who does not. Indeed, some such contrast is a commonplace. Étienne Gilson, for example, writes as follows.[1] 'There are virtually only two great roads open to metaphysical speculation: that of Plato and that of Aristotle. One can have a metaphysics of the intelligible, suspicious with regard to the sensible, whose method is mathematical, which branches out into a science of measurement; or one can have a metaphysics of the concrete, suspicious with regard to the intelligible, whose method is biological, which branches out into a science of classification.' This contrast between Plato and Aristotle seems too simple, in light of the *De Sensu*'s application of mathematics to colour. So let us look more closely at what recent commentators have said about Aristotle's antipathy to a mathematical approach.

Augustin Mansion has maintained that Aristotle neglects the use of mathematical formulae in nature, except in connection with insignificant points of

[1] Translated from *Études sur le rôle de la pensée médiévale dans la formation du système cartésien*, Paris, 1930, p. 199.

detail.[1] And Léon Robin says that mathematics is used merely as a source of examples, or to present the results of empirical analysis with an outward appearance of simplicity.[2] Neither statement seems to fit the *De Sensu*'s treatment of colour.

Other commentators go further. Not only did Aristotle omit to apply mathematics to nature. He thought it quite inappropriate to do so. Friedrich Solmsen,[3] for example, puts great stress on a passage in the *De Caelo*. At $306^{a}1$–21, Aristotle is attacking Plato's construction of matter out of triangles. Plato has used mathematical first principles in dealing with nature. But perhaps, says Aristotle, the first principles of perceptible things should be perceptible, of eternal things eternal, of perishable things perishable, and in general the first principles should be of the same kind as what falls under them. If this last statement is taken seriously, it seems to mean that the natural scientist may not take his first principles from mathematics. But can the statement be taken seriously? As early as Simplicius, we find doubts raised.[4] Simplicius points out that Aristotle says only 'perhaps', and suggests he is forced into this by the fact that matter, as Aristotle conceives it, is not an object of perception, but is none the less one of the first principles of perceptible objects. Maybe Aristotle's tentative 'perhaps' is due to an unresolved problem he raises in the *Metaphysics*, at $1000^{b}23$–9. If the first principles of perishable things were themselves perishable, he says, they would not be genuine first principles, for, being perishable, they would themselves stand in need of first principles, out of which they would arise, and into which they would perish. At any rate, whatever the reason for Aristotle's 'perhaps', the *De Caelo* statement is too tentative for us to be able to rest much weight on it.

It may, in any case, be that Aristotle has only a very limited point in mind. If one is giving an explanation of sensible phenomena, one's premisses must be subject to the test of whether they are consistent with the sensible phenomena. Plato's *Timaeus* has not observed this rule, because, so Aristotle alleges, it uses premisses which are inconsistent with the observable fact that solids made of earth can be transformed into fluids. It treats its mathematical premisses as unassailable by reference to such observable facts. If this is the burden of Aristotle's complaint, he has left himself quite free to apply mathematics to nature, just so long as his hypotheses, about (say) the mathematical ratios of black to white, are not treated as ultimate, but are checked for consistency with the observable facts.

The passage just cited from the *De Caelo* comes from a long attack on Plato's treatment of matter in the *Timaeus*. Harold Cherniss[5] has said of this attack that Aristotle's fundamental objection to Platonic matter is that it is too

[1] In *Introduction à la physique aristotélicienne*, Louvain and Paris [1st edition, 1913], 2nd edition, 1946, esp. pp. 188, 225; and in 'La physique aristotélicienne et la philosophie', printed in *Philosophie et Sciences*, Journées d'études de la Société thomiste, 1936.

[2] *La Pensée grecque*, Paris, 1923, p. 332.

[3] *Aristotle's System of the Physical World*, Ithaca, New York, 1960, pp. 259–62.

[4] Commentary on Aristotle's *De Caelo*, p. 642.

[5] *Aristotle's Criticism of Plato and the Academy*, Baltimore, 1944, e.g. pp. 123, 124, 130, 161, 164. According to Cherniss, Aristotle also believes that different sounds are irreducible qualities, not to be explained as quantitative relations (p. 158 note). Because of this belief, says Cherniss, Aristotle denies in the *De Anima* that high notes are identical with swift movements ($420^{a}31$–3). But in fact, the *De Anima* passage is very guarded. And whatever it says, it does not disagree with the view of the *De Generatione Animalium* ($786^{b}25$–$787^{b}20$) that pitch varies with variation in speed.

mathematical. Plato wrongly reduces quality to quantity. None the less, whatever Aristotle's objections may be to certain uses of mathematics, these passages do not seek to condemn any and every application of mathematics to nature.

More striking perhaps is Aristotle's statement in *Metaphysics* 995^{a}14–17. We should not demand mathematical precision in natural science, because the matter of which natural objects consist interferes, in such a way as to make precision impossible. Once again, I do not believe that this statement rules out the application of mathematics to nature. It implies only that there will be some restrictions. For example, because of the recalcitrance of the material pigments, painters cannot obtain certain colours by mixing, even though, mathematically speaking, this should have been possible. Again, though we may say that red has a ratio of 2 : 1, there are in fact several shades of red, and only one shade can have *precisely* the ratio of 2 : 1. In both ways, the application of mathematics to colour must be restricted, but the application of mathematics does not for that reason become impossible.

If we now turn to the evidence on the other side, we find that Aristotle repeatedly links four sciences, astronomy, optics, acoustics, and mechanics, with mathematics. His view of the exact nature of the link changes from time to time.[1] But one way or another he has to admit that mathematics has a large part to play in these sciences.

In the *De Sensu* we find him going further. Alongside acoustics, one of the four sciences just named, we must place colour theory. For this can be given a similar mathematical treatment. And there are hints that the same might be done in other fields too. Just as black and white can be mixed in various ratios, so too hot and cold can be mixed in a ratio of 2 : 1 or 3 : 1 (*G. et C.* 334^{b}14–16). And again Aristotle in some ways approves of Empedocles[2] for saying that bone differs from flesh because of its numerical ratio, four parts of fire to two of water.[3]

In the *De Sensu* Aristotle actually seems to be more willing than Plato to apply mathematics to colour theory. In a much-discussed passage, *Timaeus* 68B, Plato appears to be saying that one should not try to state the mathematical ratios which produce various colours. Aristotle, however, has implied that the ratios are those of the consonant intervals, namely 1 : 4, 1 : 3, 1 : 2, 2 : 3, and 3 : 4. Moreover, he has roughly correlated these ratios with purple, red, and (we may guess) green and blue, and yellow, though he has not committed himself to any particular ratio for any particular colour.[4] In spite of Gaiser's attempt[5] to find a measure of agreement between Plato and Aristotle, it looks as if Aristotle has here gone beyond Plato in his willingness to apply mathematics to natural science. We cannot accept as it stands the stereotype of Plato as one who favours the application of mathematics and of Aristotle as one who opposes it.

[1] The stages in his thought are traced out by A. Mansion, *Introduction à la physique aristotélicienne*, 2nd edition, Louvain and Paris, 1946, pp. 190–5.

[2] See *Metaph.* 993^{a}17–22; 1092^{b}17; *D.A.* 408^{a}14; 410^{a}1–6; 429^{b}16; *P.A.* 642^{a}18–23; *G.A.* 734^{b}33.

[3] For further evidence of Aristotle's willingness to use mathematics, see G. E. L. Owen, 'Aristotle', in the *Dictionary of Scientific Biography*, ed. C. C. Gillispie, vol. i (1970).

[4] See p. 300, where it is pointed out that there are perhaps ten simple ratios available, and that Aristotle has not told us which of the ten correspond to the four or five pleasantest colours.

[5] Op. cit., esp. pp. 193–5.

Testability of Aristotle's Theory

This brings me to my second question about Aristotle's method. To modern ears, the use of mathematical formulae will seem worthless, unless the mathematical suggestions can be empirically tested. Vlastos[1] has supplied a valuable analysis of early Greek theories of nature, in which he emphasizes that on the whole these theories were not formulated with sufficient precision to admit of empirical testing. Plato, in his treatment of colour, appears to go further still, claiming that it would be entirely inappropriate to subject to empirical test his explanation of how the various colours are produced (*Timaeus* 68D). There is nothing like this in Aristotle. But we should not go to the other extreme. For Aristotle shows no particular awareness of the need to formulate his theory so that it will admit of empirical testing.[2] Let us ask then, not whether he has stated the theory with a view to making it empirically testable, but whether it happens to be empirically testable.

It may seem that the theory provides us with no way of discovering what the ratios for particular colours are. Aristotle has not told us what they are in any one case. One barrier to testing will arise if the material pigments are recalcitrant in any of the ways we described earlier, on pp. 298–9. Another barrier to testing is the fact that we have not been told what it means to talk of the ratio of black to white in the original ingredients. Perhaps the black in a volume of earth and the white in the same volume of fire are to be counted as the same quantity of black and of white. But this will not enable us to discover the ratios of other colours, if we cannot obtain specimens of pure earth and pure fire, know when we have got them, and set about mixing them. In fact, it is implied in the *De Generatione et Corruptione* that the earth and fire available to us are not pure (330^b21).

In spite of this, it may seem that more indirect methods of testing could be used. To give an example, suppose one takes a pint each of red, purple, green, and blue paint, and hypothesizes the exact number of units of black and of white in each. Such a hypothesis would facilitate predictions to the effect that so much of one pint, mixed with so much of another, should yield the same shade as so much of a third mixed with so much of a fourth, provided that the materials are not recalcitrant. Suppose the original hypothesis about the number of units were corroborated by the confirmation of these predictions. One might then infer that any pint of red paint would contain, not indeed exactly the same number of units of black and white, but units of black and white in the same ratio, let us say in the ratio of three whites to one black. But how much would one then have confirmed? Not surely the historical hypothesis that at some time in the past white bodies had been mixed with black, fire for example with earth, in a proportion of 3 to 1. One's result would be compatible with quite different theories, such as that red paint reflected three units of light for every one unit it absorbed. One might continue to speak of this as being composed of three units of white to one of black. But the sense of so speaking would no longer have anything to do with the historical hypothesis of mixture.

Moreover, even if such a method could be used for confirming part of Aristotle's theory, it could hardly be used for disconfirming it. For if one's

[1] Review of Cornford's *Principium Sapientiae* in *Gnomon* xxvii (1955), 65.

[2] He thinks theories about the observable world must be rejected if they are not consistent with the observable facts (see above, p. 303), but this is not yet to think of framing theories so as to fit them for empirical testing.

predictions never came out right, this could be put down to the recalcitrance of the materials. How seriously one takes this last difficulty will depend on one's view of the nature of science. Some would say that many theories of modern science have lacked the falsifiability which Karl Popper[1] considers the hallmark of scientific theory.

Explanation of Aristotle's Oversights

I now come to my third and last question about method. We earlier discovered a series of oversights in Aristotle's mathematical treatment of colour. How are these oversights to be explained?

One explanation would be that Aristotle lacked the required mathematical competence. Gaston Milhaud wrote an influential article in 1903[2] in which he argued that Aristotle was insufficiently influenced by recent developments in mathematics, and retained the most naïve conceptions, conceptions which show him to be no true mathematician. Milhaud cites, for example, Aristotle's view of number as a discontinuous plurality of units. Such a notion ignores numbers other than the integers. The only time that Aristotle shows himself aware of recent advances is when he discusses the squaring of circles or lunules. And even then he is committed to coming out on the wrong side, and to denying the possibility of squaring lunules. For he thinks a straight line is incommensurable with a circle (*Phys.* 248^b4), or curve. In discussing what we should call the laws of motion, he assumes that the relations that hold between speed, force, and density of the medium will be simple proportionalities, or inverse proportionalities. He is not alive to the existence of other mathematical relations. In denying the actualization of infinity, Aristotle cuts himself off from the infinitesimal methods that were being elaborated. Such are Milhaud's charges, and many leading scholars since have endorsed his conclusion, for example Werner Jaeger, Léon Robin, and Jean-Marie Le Blond.[3] The last of these simply refers to Milhaud for his support. W. D. Ross uses one of Milhaud's arguments when he says, 'A better mathematician might even in the absence of evidence have noticed the possibility.' (Ross is referring to the possibility that velocity and density of medium might bear a relation to each other more complex than that of inverse proportion.[4])

Henri Carteron and Augustin Mansion use a further argument for Aristotle's unfamiliarity with mathematics, namely that he appeals to specialists for the mathematical details of his celestial system, instead of working out the details

[1] *The Logic of Scientific Discovery*, Hutchinson, 1959.

[2] 'Aristote et les mathématiques', *Archiv für Geschichte der Philosophie* 1903. Also *Les Philosophes-Géomètres de la Grèce*, Paris, 1900, pp. 358–365.

[3] Jaeger, *Aristotle, Fundamentals of the History of his Development*, Oxford, 2nd edition (translated from the German of 1923): 'Aristotle lacked the temperament and the ability for anything more than an elementary acquaintance with the Academy's chief preoccupation, mathematics' (p. 21). Robin, *La Pensée grecque*, Paris, 1923, says (p. 332) that it does not seem that Aristotle had the same mastery of mathematics as Plato. Le Blond, *Logique et méthode chez Aristote*, Paris, 1939, p. 192, calls Aristotle a mediocre mathematician. In contrast to Plato, he was not fundamentally a mathematician.

[4] Aristotle, *Physics*, a revised text with introduction and commentary, Oxford, 1936, p. 29. Cf. p. 31: the need for more complexity should have been apparent also from the fact that Aristotle is forced to admit a certain exception to his proportionalities.

himself (*Metaph.* 1073^b10–17; cf. *Cael.* 291^a29–32; $^b10$).[1] The celestial system, with its 55 spheres, yields a fund of evidence. For it appears to be full of miscalculations. Aristotle has been defended on some charges more convincingly than on others. But at least when he says that the number 55 could be reduced to 47, few commentators deny that he has made an error in elementary arithmetic.

This evidence might suggest that Aristotle's oversights in the treatment of colour are due to mathematical incompetence. But caution is needed. For Aristotle's oversights in the treatment of motion a quite different, and to my mind, convincing, explanation emerges from the studies of H. Carteron and of G. E. L. Owen.[2] I do not think this explanation can be transferred in order to explain the oversights in Aristotle's colour theory. But it does modify the picture of mathematical incompetence that many writers have drawn.

Combining the arguments of Carteron and Owen, we may say that, when Aristotle postulates certain proportional relations as holding between speed and the density of the medium, his interest is in the limiting case, where density is zero. He is not especially concerned with cases that fall short of the limit. He wants to show (*Phys.* 215^a24–$^b22$) that a vacuum is impossible. For if the medium offered no resistance to motion, the speed of moving objects would *per impossibile* bear no ratio to other speeds, but would be infinite. So long as he can show that zero density involves infinite speed, he will not worry about getting other speeds exactly right. His method in connection with other speeds is to appeal to *endoxa*, that is to opinions that are commonly accepted. For it is the method of dialectic[3] to base one's conclusion (that zero density involves infinite speed) on *endoxa*. The *endoxa* may be facts of everyday observation, for example facts about the powers of ship haulers.[4] Another *endoxon* may be the simple idea that things do stand in proportion to each other.[5] The suggestion, then, that speed varies in inverse proportion to the density of the medium is entertained because it corresponds to *endoxa*, not because Aristotle is too naïve a mathematician to think of other relations. If he had introduced exact observations concerning intermediate densities and intermediate speeds, these observations would not have been *endoxa*. And this would have gone against his whole conception of the proper method of argument. One can see how unconcerned he is with finding a precise mathematical formula to cover all the cases, and how much more interested he is in *endoxa*. For he is perfectly willing to admit an exception (*Phys.* 250^a9–19) to the rules of proportion, when everyday observation suggests that there is one.

This explanation of Aristotle's oversights in the theory of motion is a good one. But we should not expect it to account for oversights elsewhere, for example in the celestial system, or in the theory of colour. No single explanation seems to fit all these cases. And even within the theory of colour, we should perhaps

[1] Carteron concludes that Aristotle was little versed in the mathematical sciences (Budé edition of Aristotle's *Physics*, 1926, vol. i, p. 16), Mansion that he neglected them ('La physique aristotélicienne et la philosophie', op. cit. (1936), pp. 26–7; cf. *Introduction à la physique aristotélicienne*, 2nd edition, 1946, p. 188).

[2] H. Carteron, *La Notion de force dans le système d'Aristote*, Paris, 1924; G. E. L. Owen, 'Aristotle', in *Dictionary of Scientific Biography*, ed. C. C. Gillispie, vol. i (1970).

[3] See the definition of dialectic, at *Top.* 100^a29–30, as reasoning that starts from *endoxa*.

[4] For appeal to these facts, but as providing an exception, not confirmation, see *Phys.* 250^a17–19.

[5] See *Phys.* 250^a3–4, 'for in this way there will be a proportion.'

not expect a single explanation for the five oversights we have catalogued. Some of these oversights (the first and fifth) scarcely require explanation, the first because it is a mere omission, the fifth because it is not of Aristotle's own making. The fourth oversight, whereby he postulates a single string-length ratio for the octave and the fifth, could be a carelessness, fostered by his lack of interest in an argument which he rejects. This leaves us with two oversights which are harder to explain.

When he fails to notice that there are ten simple ratios, too many to explain the scarcity of pleasant colours, he has perhaps fallen victim to his willingness to leave mathematical details to people more expert than himself. Just as he says that he will leave to specialists the mathematical details of his celestial system (*Metaph.* 1073^b10–17, cited above), so here there are signs that he is leaving to others the details of his colour theory. For three times he gives us alternative hypotheses, without firmly deciding between them.[1]

But this will not account for what seems to be the worst oversight, the neglect of the middle ground between the simplest ratios and the irrational ones. Followers of Milhaud will be quick to conclude that Aristotle was unfamiliar with the facts of acoustical theory. It seems hard to believe that he could have been unfamiliar with facts so elementary, and yet it is also hard to avoid this conclusion.

RETROSPECT

There has been a stereotype of Aristotle as differing from Plato in being unwilling to apply mathematics to science. What we find in his theory of colour is not an unwillingness at all, but instead a great deal of oversight in the details of the application. These oversights have not one explanation, but a variety of different ones.[2]

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[1] 440^a3; 442^a22; see p. 296 n. 3 on 442^a14–16.

[2] I read earlier drafts of this paper in three places, and received many helpful comments. I have responded to, or made use of, those by Professor J. L. Ackrill, by Jonathan Barnes, by Willie Charlton, and by Professor H. Post and his colleagues at the Chelsea College of Science and Technology. The donkey-work on Aristotle's *De Sensu* was done while I held a Howard Foundation Fellowship from Brown University, and a project grant (no. H68-o-95) from the National Endowment for the Humanities.

I have used the following abbreviations:

An. Post.	*Analytica Posteriora*
Top.	*Topica*
Phys.	*Physica*
Cael.	*De Caelo*
G. et C.	*De Generatione et Corruptione*
Meteor.	*Meteorologica*
D.A.	*De Anima*
D.S.	*De Sensu*
G.A.	*De Generatione Animalium*
Probl.	*Problemata*
Metaph.	*Metaphysica*